

Frequency Block Hopping in the Uplink of OFDM-based Wireless Systems

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Abstract: In this paper, we consider the use of frequency block hopping in the uplink of orthogonal frequency division multiplexing (OFDM) based wireless systems. The hopping block size influences the channel estimation accuracy and thus affects the effective signal-to-noise power ratio (SNR) of the receiver as well as the diversity gain. The hopping block size is optimally determined to minimize the average error probability which is associated with the SNR and diversity gain. Simulation results show that the analytic design is practically applicable to various channel coding schemes.

Keywords: frequency hopping, OFDM, uplink.

1. INTRODUCTION

In recent years, orthogonal frequency division multiplexing (OFDM) has attracted a great attention as one of the best transmission techniques for next generation mobile communication systems [1]. It can provide high spectral efficiency by converting frequency selective fading into a series of narrowband flat fading [3]. Especially, it can achieve a diversity gain with the use of frequency hopping techniques [4].

In the downlink, the base station (BS) generally transmits a common pilot signal that spans the whole channel bandwidth, enabling the mobile station (MS) to estimate the channel condition [5]. In this case, the BS can employ a symbol frequency hopping technique where the transmission block is divided into several symbols, being scattered on the whole channel bandwidth. In the uplink, however, it may not be feasible for each MS to transmit such a common pilot signal in practice [5]. To alleviate the channel estimation problem, the use of a frequency block hopping technique is often considered in the uplink of OFDM systems [6], where the transmission block comprises several hopping blocks each of which contains data and pilot symbols together.

The hopping block size can affect on the diversity gain due to the frequency hopping and the SNR degradation due to the channel estimation error. The smaller the hopping block size, the larger the diversity gain and also the channel estimation error (i.e., SNR degradation) due to the increase of the number of hopping blocks and the decrease of the number of pilot symbols in the hopping block, respectively, and vice versa. As a consequence, there is a trade-off issue between the diversity gain and the SNR degradation associated with the hopping block size. It may be desirable to optimize the hopping block size that minimizes the average error probability. To this end, we first analyze the average error probability associated with the SNR degradation and diversity gain according to the hopping block size, assuming the use of a simple repetition code. Then, we determine the optimum hopping block size that

minimizes the average error probability.

The rest of the paper is organized as follows. We describe the system model in consideration in Section II. We analyze the average error probability in terms of the hopping block size and then determine the optimum hopping block size in Section III. We verify the analytic results by computer simulation in Section IV and finally summarize conclusions in Section V.

2. SYSTEM MODELING

We describe the uplink model of an OFDM-based wireless system that utilizes frequency block hopping. We assume that each MS is allocated to a transmission block comprising N_t OFDM symbols and N_f subcarriers as shown in Fig. 1. We also assume that the hopping block size is regular in the time domain (i.e., the transmission block is divided into K ($= N_f/u$) hopping blocks as illustrated in Fig. 2), where u , T_s and Δf denote the hopping block size in the frequency domain, the OFDM symbol duration, and the subcarrier spacing, respectively. Each hopping block has a span of $N_t T_s$ and $u \Delta f$ in the time and frequency domain, respectively.

Let $\mathbf{h}^{(k)}$ be the $(N_t \times u)$ channel matrix of the k -th hopping block defined as

$$\mathbf{h}^{(k)} = \begin{bmatrix} h_{11}^{(k)} & h_{12}^{(k)} & \cdots & h_{1u}^{(k)} \\ h_{21}^{(k)} & h_{22}^{(k)} & \cdots & h_{2u}^{(k)} \\ \vdots & \vdots & \ddots & \vdots \\ h_{N_t 1}^{(k)} & h_{N_t 2}^{(k)} & \cdots & h_{N_t u}^{(k)} \end{bmatrix} \quad (1)$$

where $h_{ij}^{(k)}$ denotes the channel impulse response (CIR) at the i -th OFDM symbol and the j -th subcarrier of the k -th hopping block. Assuming that $N_t T_s$ and $u \Delta f$ are smaller than the coherence time and bandwidth, respectively, it can be assumed that the hopping block experiences flat fading. Then, all the elements of $\mathbf{h}^{(k)}$ are the same as $h^{(k)}$ which is defined as the representative channel of the k -th hopping block.

Let x_{ij} be the transmitted signal and w_{ij} be additive white Gaussian noise (AWGN) with variance σ_w^2 at the i -th OFDM symbol and the j -th subcarrier of the transmission block. Then, the received signal matrix of transmission block can be expressed as

$$\mathbf{Y} = \mathbf{H} \oslash \mathbf{X} + \mathbf{W} \quad (2)$$

where \oslash denotes the element-by-element product of two matrices, \mathbf{H} is the $(N_t \times N_f)$ channel matrix of a transmission block defined as

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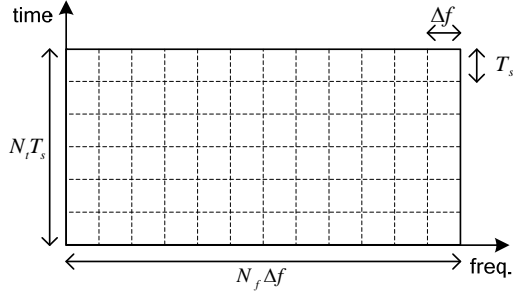


Fig. 1. Transmission block.

$$\mathbf{H} = [\mathbf{h}^{(1)} \mathbf{h}^{(2)} \dots \mathbf{h}^{(k)} \dots \mathbf{h}^{(K-1)} \mathbf{h}^{(K)}] \quad 3$$

\mathbf{X} is the $(N_t \times N_f)$ transmitted signal matrix defined as

$$\mathbf{X} = \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1N_f} \\ x_{21} & x_{22} & \dots & x_{2N_f} \\ \vdots & \vdots & \ddots & \vdots \\ x_{N_t 1} & x_{N_t 2} & \dots & x_{N_t N_f} \end{bmatrix} \quad 4$$

and \mathbf{W} is the $(N_t \times N_f)$ noise matrix defined as

$$\mathbf{W} = \begin{bmatrix} w_{11} & w_{12} & \dots & w_{1N_f} \\ w_{21} & w_{22} & \dots & w_{2N_f} \\ \vdots & \vdots & \ddots & \vdots \\ w_{N_t 1} & w_{N_t 2} & \dots & w_{N_t N_f} \end{bmatrix} \quad 5$$

Assume that the pilot symbol is regularly placed at every $(d_t + 1)$ OFDM symbols and $(d_f + 1)$ subcarriers in each hopping block. Let M_t and M_f be the number of pilot symbols in the time and frequency domain, respectively, where $M_t = \lfloor N_t / (d_t + 1) \rfloor + 1$, $M_f = \lfloor N_f / (d_f + 1) \rfloor + 1$. Here, $\lfloor A \rfloor$ denotes the smallest integer less than or equal to A .

AVERAGE ERROR PROBABILITY

We optimize the hopping block size minimizing the average error probability. To this end, we first investigate the channel estimation error due to the change of the hopping block size, and then its effect on the SNR degradation. We derive the probability density function (PDF) of the channel gain in terms of the hopping block. Finally, the average error probability is analyzed in terms of the SNR degradation and the PDF.

A. Channel Estimation Error

We assume that the BS estimates the CIR at the pilot symbol by using a least square estimator in the frequency-domain [7]. Then the CIR at the data symbol can be estimated from the estimated CIR at the pilot symbol. Since the hopping block experiences flat fading, the CIR at the data symbol can be estimated by averaging the estimated CIR at the pilot symbol.

The representative channel of the k -th hopping block can be estimated as

$$\hat{h}^{(k)} = \frac{1}{M_t M_f} \sum_{i_p=1}^{M_t} \sum_{j_p=1}^{M_f} \tilde{H}^{(k)}(i_p, j_p) \quad 6$$

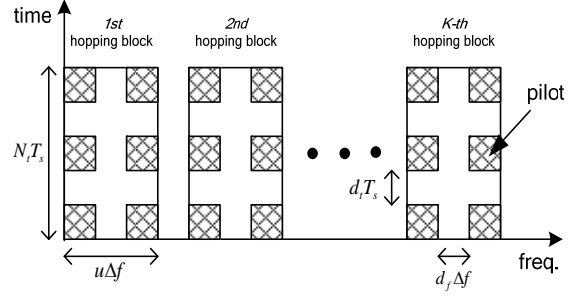


Fig. 2. Frequency block hopping in uplink.

where $\tilde{H}^{(k)}(i_p, j_p)$ denotes the estimated CIR at the i_p -th pilot symbol in the time domain and the j_p -th pilot symbol in the frequency domain of the k -th hopping block. The corresponding mean square error of the channel estimation can be represented as [8]

$$\begin{aligned} \varepsilon^2 &= E \left\{ \left| h^{(k)} - \hat{h}^{(k)} \right|^2 \right\} \\ &= \sigma_w^2 / (M_t M_f) \\ &= \sigma_w^2 / \left(M_t \left(\left\lfloor u / (d_f + 1) \right\rfloor + 1 \right) \right) \end{aligned} \quad 7$$

The estimated channel matrix can be represented as

$$\hat{\mathbf{H}} = \mathbf{H} + \varepsilon \mathbf{Z} \quad 8$$

where

$$\mathbf{Z} = [\mathbf{z}^{(1)} \mathbf{z}^{(2)} \dots \mathbf{z}^{(k)} \dots \mathbf{z}^{(K)}] \quad 9$$

and $\mathbf{z}^{(k)}$ is an $(N_t \times u)$ matrix whose entries are zero-mean complex Gaussian random variables with the same unit variance.

B. SNR Degradation

Assume that the data is encoded using a repetition code with code rate $1/N_f$, each data symbol is transmitted through an OFDM symbol. Then, the received signal vector of the q -th OFDM symbol can be represented as

$$\mathbf{Y}_q = \mathbf{H}_q \mathbf{s}_q + \mathbf{W}_q, \quad q = 1, 2, \dots, N_t \quad 10$$

where \mathbf{A}_q denotes the q -th row vector of matrix \mathbf{A} and \mathbf{s}_q is the q -th data symbol. Based on estimated $\hat{\mathbf{H}}_q$, the BS can decode \mathbf{s}_q by employing a maximum ratio combiner (MRC) [3]. The output of the MRC can be represented as

$$r_q = \mathbf{Y}_q \frac{\hat{\mathbf{H}}_q^H}{\|\hat{\mathbf{H}}_q\|} = \frac{\|\mathbf{H}_q\|^2 \mathbf{s}_q}{\|\hat{\mathbf{H}}_q\|} + \frac{\varepsilon \mathbf{H}_q \mathbf{Z}_q^H \mathbf{s}_q}{\|\hat{\mathbf{H}}_q\|} + w' \quad 11$$

where the superscript H denotes conjugate transpose and w' denotes AWGN with variance σ_w^2 . The second term in (11) represents the channel estimation error, yielding SNR degradation. For a given \mathbf{H}_q , the signal power P_s and the noise power P_w of the received signal can respectively be represented as

$$P_s = E \left\{ \left| \frac{\|\mathbf{H}_q\|^2 s_q}{\|\hat{\mathbf{H}}_q\|^2} \right|^2 \right\} = \frac{\sigma_s^2 \|\mathbf{H}_q\|^4}{\|\mathbf{H}_q\|^2 + N_f \varepsilon^2} \quad (12)$$

$$P_w = E \left\{ \left| \frac{\varepsilon \mathbf{H}_q \mathbf{Z}_q^H s_q}{\|\hat{\mathbf{H}}_q\|^2} + w' \right|^2 \right\} \simeq \varepsilon^2 \sigma_s^2 + \sigma_w^2 \quad (13)$$

where $\sigma_s^2 = E \left\{ |s_q|^2 \right\}$.

The effective SNR of the received signal can be estimated as

$$\begin{aligned} \bar{\gamma}(\lambda_q) &= \frac{\sigma_s^2 \|\mathbf{H}_q\|^4}{\|\mathbf{H}_q\|^2 + N_f \varepsilon^2} \\ &\simeq \frac{\lambda_q^2}{(\lambda_q + N_f \varepsilon^2)(\varepsilon^2 + (\sigma_w^2 / \sigma_s^2))} \end{aligned} \quad (14)$$

where $\lambda_q (= \|\mathbf{H}_q\|^2)$ represents the channel gain. Letting $\varepsilon^2 = 0$, the ideal SNR is represented as

$$\bar{\gamma}_{id}(\lambda_q) = \frac{\lambda_q \sigma_s^2}{\sigma_w^2} \quad (15)$$

Define the SNR degradation due to imperfect channel estimation by

$$\begin{aligned} \Delta \bar{\gamma}(\lambda_q) &\triangleq \bar{\gamma}(\lambda_q) - \bar{\gamma}_{id}(\lambda_q) \\ &= \frac{\lambda_q^2}{(\lambda_q + N_f \varepsilon^2)(\varepsilon^2 + (\sigma_w^2 / \sigma_s^2))} - \frac{\lambda_q \sigma_s^2}{\sigma_w^2}. \end{aligned} \quad (16)$$

C. Average Error Probability

The diversity gain due to the combining can be represented in terms of the PDF of the channel gain. Since the number of hopping blocks is associated with the hopping block size, the diversity gain can be represented in terms of the hopping block size.

The channel gain of the received signal can be represented as

$$\lambda_q = \|\mathbf{H}_q\|^2 = u \sum_{k=1}^K |h^{(k)}|^2 \quad (17)$$

Since $h^{(k)}$ is a Gaussian random variable, λ_q is Chi-square distributed with $2K$ degrees of freedom and the corresponding PDF is given by [9]

$$f_\lambda(\lambda_q) = \frac{1}{u(N_f/u - 1)!} \left(\frac{\lambda_q}{u} \right)^{N_f/u - 1} e^{-\left(\frac{\lambda_q}{u} \right)} \quad (18)$$

where $\lambda_q \geq 0$. The corresponding average error probability can be calculated by [3]

$$P_e = \int_0^\infty f_\lambda(\lambda_q) V(\bar{\gamma}(\lambda_q)) d\lambda_q \quad (19)$$

where $V(\bar{\gamma}(\lambda_q))$ is the error probability function according to

TABLE I. Simulation Parameters

Parameters	Values
Carrier frequency	5.8 GHz
Subcarrier spacing	46.875 KHz
OFDM symbol duration	20.48 us
Guard interval	5 us
Channel	Rayleigh fading
Modulation	16 QAM
Transmission block	(8,16)
Pilot spacing	(5,3)

the modulation scheme.

The average error probability according to the hopping block size can be modified by using (14) and (18). When binary phase shift keying (BPSK) is used as the modulation scheme, the error probability function can be represented as [10]

$$V(\gamma(\lambda_q)) = Q \left(\sqrt{\frac{2\lambda_q^2}{(\lambda_q + N_f \varepsilon^2)(\varepsilon^2 + (\sigma_w^2 / \sigma_s^2))}} \right) \quad (20)$$

where

$$Q(x) = \frac{1}{2\pi} \int_x^\infty e^{-t^2/2} dt \quad (21)$$

The average error probability can be calculated by

$$P_e = \int_0^\infty Q \left(\sqrt{\frac{2\lambda_q^2}{(\lambda_q + N_f \varepsilon^2)(\varepsilon^2 + (\sigma_w^2 / \sigma_s^2))}} \right) \frac{1}{u(K-1)!} \left(\frac{\lambda_q}{u} \right)^{K-1} e^{-\left(\frac{\lambda_q}{u} \right)} d\lambda_q. \quad (22)$$

When modulation schemes other than BPSK are employed, the average error probability can be calculated by modifying (20).

. OPTIMUM HOPPING BLOCK SIZE

It is not easy to explicitly obtain the optimum size from (22) due to involved nonlinear functionality. However, since the hopping block size is an integer divisor of N_f , the optimum hopping block size that minimizes the average error probability can be found by simulation without difficulty. As a design example, we consider a system with parameters summarized in Table I [11].

depicts the SNR degradation due to the channel estimation error in terms of the hopping block size. It can be seen that the SNR degradation increases as the hopping block size decreases due to the decrease of pilot symbols in the hopping block. This means that the use of a larger hopping block size is preferred for the achievement of higher SNR. It can also be seen that the SNR is more affected by the hopping block size as the SNR decreases. This is mainly due to the fact that the channel estimation error is mostly affected by the additive noise.

depicts the PDF of the channel gain for different hopping block sizes. It can be seen that as the hopping block size decreases, the variance of the channel gain decreases, reducing the effect of fading. This implies that the use of a smaller hopping block size is desirable for the diversity gain. It can be seen from Fig. 3 and 4 that there is a trade-off issue between the diversity gain and the SNR degradation associated with the hopping block size. Thus, it may be desirable to determine the hopping block size by jointly considering the diversity gain and SNR degradation.

depicts the average error probability due to the size of the hopping block with the use of repetition and convolutional channel codes in various SNR condition. It can be seen that the

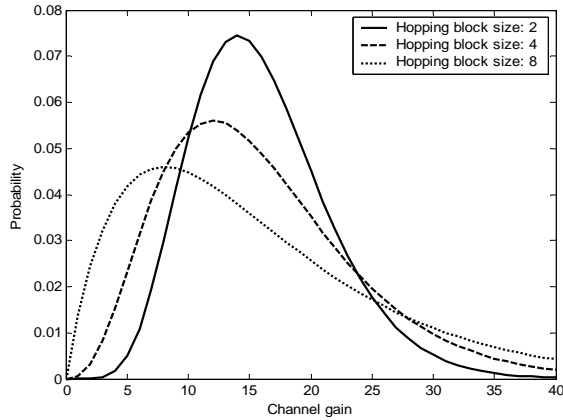


Fig. 3. SNR degradation according to the hopping block size.

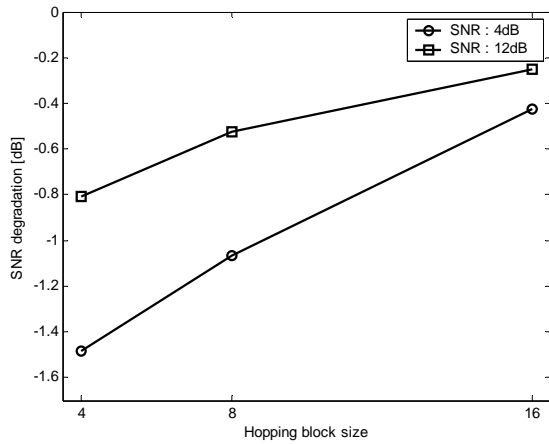
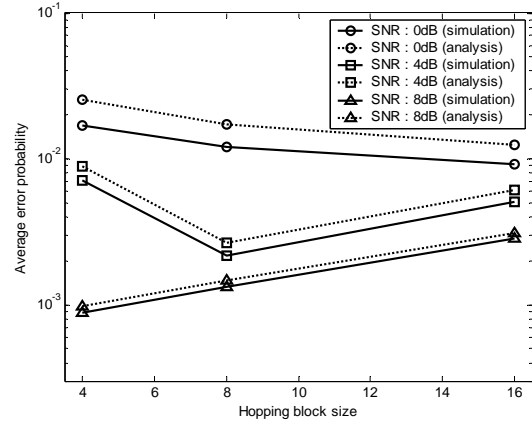


Fig. 4. SNR degradation according to the hopping block size.

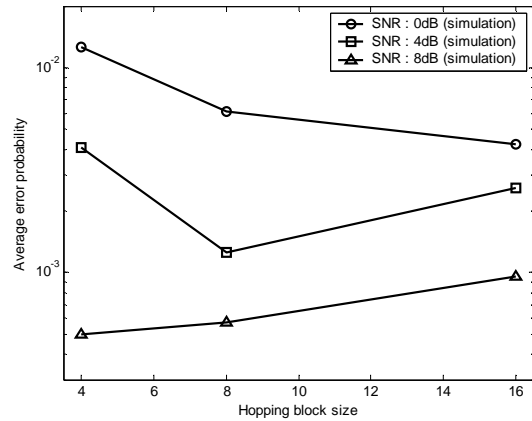
optimum hopping block size can be found to minimize the average error probability at a given SNR and that the analytic results quite agree well with the simulation results. It can be seen that the use of a smaller hopping block size is desirable when the SNR is high, and vice versa. This is mainly due to the fact that the smaller the hopping block, the larger the diversity gain in high SNR environments, while SNR degradation due to the channel estimation error is insignificant. On the other hand, when the SNR is low, it is desirable to increase the hopping block size to improve the estimation performance. It can also be seen that the optimum hopping blocks size is somewhat indifferent from the channel coding scheme.

. CONCLUSIONS

In this paper, we have considered the optimization of the hopping block size in the uplink of OFDM-based wireless systems. The optimum size can be determined to minimize the average error probability considering the trade-off between the SNR degradation and the diversity gain associated with the hopping block size. Analyzing the average error probability, we have optimized the hopping block size that minimizes the average error probability.



(a) Repetition code



(b) Convolutional code

Fig. 5. Average error probability according to the hopping block size.

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